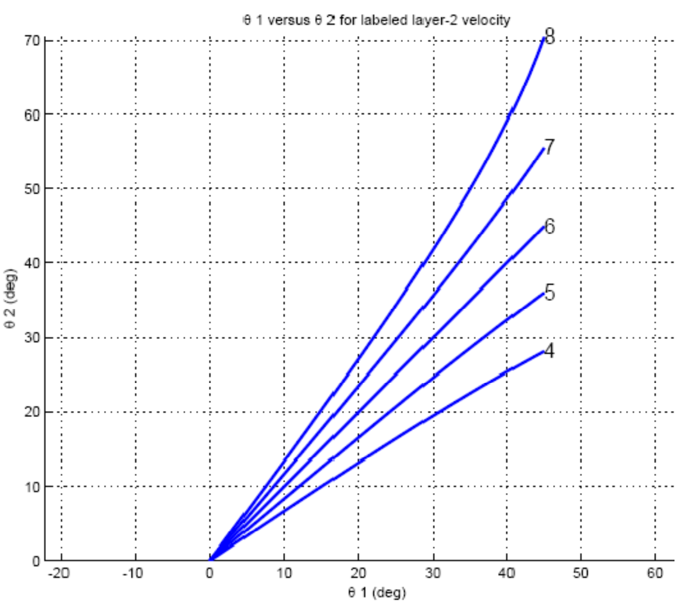
Introduction Geophysics Lab: Snell's Law Derivation Name________________________

$$\frac{\sin(\theta_1)}{v_1} = \frac{\sin(\theta_2)}{v_2} \qquad \frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{v_1}{v_2} = \frac{\lambda_1 * f}{\lambda_2 * f} \qquad v_i = \lambda_i / T = \lambda_i * f \quad f = 1/T$$

where the 1 subscript refers to the upper layer in which the wavefront (ray) are incident and the 2 subscript refers to the lower layer in which the transmitted wavefront (ray) is refracted. v_i is the i'th layer velocity (m/s) and λ_i (m) is the wave's wavelength in the i'th layer and f is frequency (s^{-1}) and T (s) is wave period. Snell's Law simply states the equality between ratios of the sine of the incident angle divided by the layer velocity. Algebraic manipulation also shows the equality of the adimensional ratios of sin(angle), velocities, and wavelengths. Important is to understand that the frequency does NOT (and CANNOT) change in this physics formulation: therefore, for a given frequency, the higher the velocity of an elastic medium, the longer the wavelength; the lower the velocity of an elastic medium, the shorter the wavelength. This change in wavelength when a wave crosses an interface that has a change in the rocks velocity, means that the wave's wavelength must change (frequency does not change). For this change in wavelength to be accommodated without 'tearing' apart the wavefield continuum, the wave must refract. Refraction is a wave phenomenon that all waves (E-M, Seismic) must obey (non negotiable point).

1. (3 pt) Assuming a wave front (ray) is traveling downwards in layer-1 and meets a velocity interface which represents the boundary between layer-1 (V_1) and layer-2 (V_2). Below is a plot of incident angle (Θ_1) and the refracted wave angle (Θ_2) where the layer-2 velocity is listed by the number 4-8 km/s in the middle of the plot. (a) Assume that the 6 km/s layer-2 velocity straight line graph (Θ_1=Θ_2), is observed in the lab, using Snell's Law and this graph, calculate the velocity of layer-1. (b) With respect to the normal to the interface, if $v_2 > v_1$, demonstrate mathematically if the transmitted wave refracts towards or away from the interface normal? Do this same analysis for the case of $v_2 < v_1$. (c) Label the curves where $v_2 < v_1$ and $v_2 > v_1$.



2. (5 pt) Assume a layer-1 with velocity v_1 that overlies layer-2 with velocity v_2 where $V_2 > V_1$ with an incident wavefront whose angle of incidence with respect to the interface normal is Θ_1. (a) Do a complete and well-presented derivation of Snell's law using two right triangles that share a common hypotenuse. Include a clean hand-drawn figure that has the angles and right-triangle lengths clearly labeled. Label the two wavefronts at time t_0 and t_1 and the wavefronts two associated raypaths used in the derivation. (b) Describe in words each step of the derivations; this clear writing will be half the problem grade. This means you should re-write the derivation a few times to get the presentation of the combined figure, mathematics, and words in logical order. You will do this derivation on the next test. *Hint*: draw two wavefronts at time t_0 and t_1 that refract (change angle) across the velocity interface; we may choose the two times so that the time interval $(t_1 - t_0)$ is one wave period (T (s)) hence the distance between the two times is one wavelength.

3. (3 pt) (a) For Snell's Law, Why is the domain of the sine function from -90 to +90 degrees (–pi/2 to pi/2 radians)?
(b) What value is calculated if one attempts to take the inverse sine of a value outside the -1 to +1 real interval?
(c) Assume two layers with velocity v_1 and v_2 and that the refracted (transmitted) wave angle Θ_2 = 90°, solve for the incident ray angle Θ_1 (this will become a new wave type in the future).
(d) Solve the implicit transcendental equation sin(x)/a = sin(y)/a for a function y=f(x).